Basic Project Finance:
Shortcuts to NPV and DPB

Version 1.0.0 - Johannes Aurich*

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I've been involved in techno-economic analyses of industrial equipment projects recently, and this document is meant as a personal reminder of key concepts. It also documents the back-of-a-napkin approaches for present value and discounted payback time I derived to simplify the early-stage comparative analysis of project configurations. Rigorous mathematical definition and notation is not applied.

Nominal vs. Real Financial Value

Simply said, nominal financial values are those printed on receipts and bank notes, stated on accounts, and usually used to calculate taxes. They represent the exchange value of goods and services in a specific year. Real values, on the other hand, reflect the actual purchasing power of money. They consider that inflation erodes monetary value over time and express how much a nominal value in a future year t is worth relative to a base year $t = 0$. Assuming a constant inflation rate π over a time period t, the real value $V_{r,t}$ of a nominal financial value $V_{n,t}$ in year t is defined as:

$$V_{r,t} := \frac{V_{n,t}}{(1+\pi)^t} \tag{1}$$

As can be seen in equation 1, nominal and real financial values are identical in the base year and inflation effects are zero. Real values and inflation rates are always defined relative to the base year, which is often chosen as the present day in most calculations but can generally be any other reference year depending on the context. Real-world inflation rates typically vary over time and may require more complex modeling approaches than discussed here.

Present Value

The present value PV is a key financial metric for investment projects and usually applied to cash flows. It can be defined in real or nominal terms, however nominal values more typical because of easier accessibility and more straightforward use. It adjusts or, more commonly, discounts a nominal cash flow $C_{t,n}$ in year t by a nominal discount rate d_n relative to a base year $t = 0$:

$$PV := \frac{C_{t,n}}{(1+d_n)^t} \tag{2}$$

Equation 2 shows that this is conceptually related to real and nominal values. However, beyond adjusting for inflation, the discount rate usually includes additional premiums for project risk, time value of money, and financing costs. This means the present value is often lower than the real monetary value, providing a more realistic value assessment of future financial transactions. The discount rate is often calculated using the concept of WACC (weighted average cost of capital), which won't be further discussed here. Adapted to the specific context, the present value concept is widely applied across industries.

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If instead of a single cash flow a series of cash flows over multiple years must be considered, the overall present value is obtained by summing the present value of all individual cash flows:

$$PV = \sum_{t=0}^{T} \frac{C_{t,n}}{(1+d_n)^t} = \sum_{t=0}^{T} PV_t \tag{3}$$

This additive property is very useful if we consider that every cash flow of a year may itself be the sum of multiple cash flows. For example, the concept of the **Net Present Value (NPV)** defines each cash flow as the sum of all revenues $R_{t,n} > 0$ and all expenses $E_{t,n} < 0$ in that year: $C_{t,n} := R_{t,n} + E_{t,n}$. We can then determine the NPV by calculating and then summing the present values of revenues and expenses separately:

$$NPV := \sum_{t=0}^{T} \frac{R_{t,n} + E_{t,n}}{(1+d_n)^t} \tag{4}$$

$$NPV = \sum_{t=0}^{T} \frac{R_{t,n}}{(1+d_n)^t} + \sum_{t=0}^{T} \frac{E_{t,n}}{(1+d_n)^t} \tag{5}$$

$$= \sum_{t=0}^{T} PV(R_{t,n}) + \sum_{t=0}^{T} PV(E_{t,n}) \tag{6}$$

This allows to single out and treat individual value contributors separately and, depending on the problem, simplify the overall approach. For example, a machine may require the replacement of key components as an overhauling investment in years $t = (5, 8)$, additional to the annual operating expenses. Multiple service providers and spare part vendors shall be compared. A simple approach is to calculate the present value of all candidate offers (equation 3: $PV = PV_5 + PV_8$) and then choose the most economic one. No other cashflows must be considered.

A Simplified Model for Industrial Investment Projects

The scenario I consider in the following is idealized and only suitable for early-stage screenings of different project configurations. It is based on three assumptions:

1. **Commercial operation** takes place in every year $t \geq 1$ until the end of life in year $t = T$ is reached. No operation happens in base year $t = 0$ where the investment happens. This reflects the case of machinery requiring significant time for delivery, installation, and commissioning after purchase or software being implemented and users trained before roll out. The equations shown below can be adapted in case other scenarios require different time intervals.

2. **All recurring real operational cash flows are constant over time**. This is based on the premise that the intrinsic value of inputs and outputs, calculated from first principles of the machine's operation, does not change in a stable market environment. Inflation may change nominal cash flows over time, but these effects are typically passed on to customers and preserve real cash flow stability. This assumption also implies the asset operates identically every year, either neglecting degradation or assuming maintenance to mitigate its effects. If non-constant cash flows (due to degradation or other factors) must be considered, alternative approaches are discussed below or annuity calculations may be applied to calculate a constant periodic value."

3. **Constant annual rates** for inflation and general discounting are assumed.

Sign Convention: Cash flows leaving company accounts are considered < 0 (costs); cash flows into company accounts > 0 (revenues).

Remembering that we can split present value sums into the sum if it's constituents, we may calculate the present value of a project PV_{Pr} as follows,

$$PV_{Pr} = PV_{In} + PV_{Op} \tag{7}$$

with the present value of all investments PV_{In} and of all operational cash flows PV_{Op} over the project lifetime.

Investments PV_{In}

Investments shall be defined as non-recurring cash flows and can either be a single CAPEX payment in t=0, in that case $PV_{In} = C_0$, or related to a staged deployment where multiple cash flows are handled according to equation 3. Investments related to replacement and overhauling are treated similarly.

Operational cash flows PV_{Op}

Operational cash flows are recurring in every time period $t \geq 1$. We assumed above that all changes in operational cash flows are only due to inflation and the real values stays constant. This can be written as:

- Nominal cash flows are equal to inflated real cash flows (equation 1): $C_{t,n} = C_{t,r}(1+\pi)^t$.
- Every real operating cashflow is equal to a constant value: $C_{t,r} = \bar{C}_r$.
- Combined, this yields $C_{t,n} = \bar{C}_r(1+\pi)^t$

Substituting in equation 3, we can rearrange to

$$PV_{Op} = \sum_{t=1}^{T} \frac{\bar{C}_r(1+\pi)^t}{(1+d_n)^t} \tag{8}$$

$$= \bar{C}_r \sum_{t=1}^{T} \frac{(1+\pi)^t}{(1+d_n)^t} \tag{9}$$

$$= \bar{C}_r \sum_{t=1}^{T} k^t \tag{10}$$

and introduce a **Discount Factor** k^t. This parameter is equal for nominal and real discounting approaches; both can be converted into each other using the Fisher Equation $(1+d_n) = (1+d_r)(1+\pi)$:

$$k = \frac{(1+\pi)}{(1+d_n)} = \frac{1}{(1+d_r)} \tag{11}$$

Summing the discount factors for all periods in the project lifetime T gives the **Cumulative Discount Factor** CDF:

$$CDF = \sum_{t=1}^{T} k^t \tag{12}$$

This factor can be calculated directly, e.g. by using =SUM(k^SEQUENCE(T)) in Microsoft Excel. Alternatively, a solution can be found if one recognizes the similarity to the finite geometric series, which has an explicit solution[1]:

$$\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a} \quad \text{for} \quad a \neq 1 \tag{13}$$

[1] Merziger et al.: Formeln + Hilfen Höhere Mathematik, 6. Edition. Binomi Verlag, Barsinghausen, 2010

The finite geometric series includes terms from $t = 0 \dots T$, while the CDF in this scenario only ranges from $t = 1 \dots T$. This is easily solved by subtracting the summand k^t at $t = 0$, which is $k^0 = 1$:

$$CDF = \sum_{t=1}^{T} k^t = \left(\sum_{t=0}^{T} k^t - k^0 \right) = \frac{1 - k^{T+1}}{1 - k} - 1 \tag{14}$$

Substituting in equation 10, we get the simple present value formula:

$$PV = PV_{In} + \bar{C}_r \cdot CDF \tag{15}$$

Non-financial present value

The present value is not restricted to cash flows. One can equally calculate a present value of material, energy, or other quantities that are relevant to judge the performance of a process or machine, using the exact same formulas. A typical application is the levelized costs of energy (LCOE) concept[2].

Degradation and cost escalation

It has been assumed above that all periodically recurring values, no matter if monetary or not, are constant in all years. However, an additional factor can model degradation of machinery outputs and related revenues or any regular price escalation. Under the assumption that this change occurs with a constant annual percentage rate $-1 < c < 1$, it can be written:

$$C_{t,r} = \bar{C}_r (1 + c)^t \tag{16}$$

The discount factor can be adapted accordingly to equation 8 so that:

$$k = \frac{(1 + \pi)(1 + c)}{(1 + d_n)} = \frac{(1 + c)}{(1 + d_r)} \tag{17}$$

Mind that degradation or decrease in general means $c < 0$ and increase is $c > 0$.

Discounted pay back time

Discounted Payback Time τ is a financial metric used to determine the time it takes for an investment to pay back its cost while accounting for the changing value of cash flows over time. It is defined as the number of periods when the net present value of a project becomes equal to zero:

$$NPV(\tau) = 0 = \sum_{t=0}^{\tau} \frac{C_{t,n}}{(1 + d_n)^t} \tag{18}$$

We can rewrite using equations 14 and 15:

$$0 = PV_{In} + \bar{C}_r \cdot CDF \tag{19}$$

$$\frac{-PV_{In}}{\bar{C}_r} = \left(\frac{1 - k^{\tau+1}}{1 - k} - 1 \right) \tag{20}$$

Mind that an NPV is calculated here and therefore $\bar{C}_r$ represents a constant sum of all annual revenues and expenses, as shown in equation 4. Rearranging gives

$$k^{\tau+1} = 1 - \left((1 - k) \left(1 - \frac{PV_{In}}{\bar{C}_r} \right) \right) \tag{21}$$

and applying logarithms and further rearrangement finally leads to an explicit equation for τ within the frame of this document's scenario:

$$\tau = \frac{ln \left[1 - \left((1 - k) \left(1 - \frac{PV_{In}}{\bar{C}_r} \right) \right) \right]}{ln(k)} - 1 \tag{22}$$

[2]https://www.uni-mannheim.de/media/Einrichtungen/mises/Dokumente/Levelized_Cost_Review.pdf